Fluctuations of conserved charges within dynamic models of heavy-ion collisions

Marlene Nahrgang

SUBATECH, Nantes & FIAS, Frankfurt

FCR workshop, BNL, October 3rd, 2011

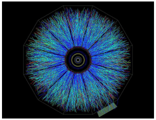
How to study the QCD phase diagram...

... be brave and solve

$$Z(T,\mu_B) = \int \mathcal{D}(A,q,q^\dagger)\mathrm{e}^{-S^E_{\mathrm{QCD}}}$$

ab initio and nonperturbatively,

... be strong and collide heavy ions at ultrarelativistic energies,



... be creative and study effective models of QCD.

$$\mathcal{L}_{\mathrm{eff}}$$

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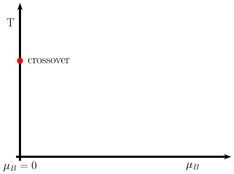

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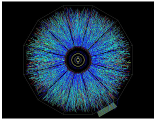

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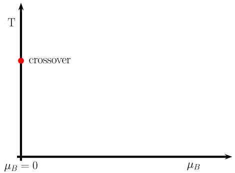

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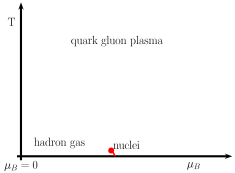

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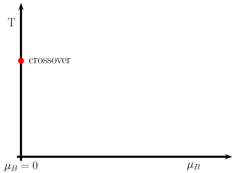

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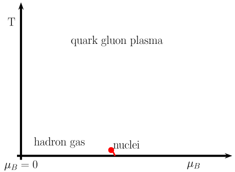

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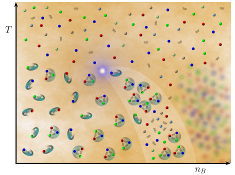
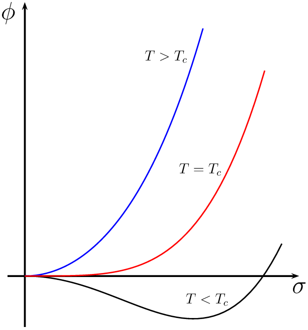


Critical point

- $m_\sigma^2 = \frac{\partial^2 V}{\partial \sigma^2} \to 0$
- correlation length diverges
  $\xi = \frac{1}{m_\sigma} \to \infty$
- universality classes
  for QCD: $\mathcal{O}(4)$ Ising model in 3d $\Rightarrow \langle \sigma^2 \rangle \propto \xi^2$
- renormalization group
- critical opalescence

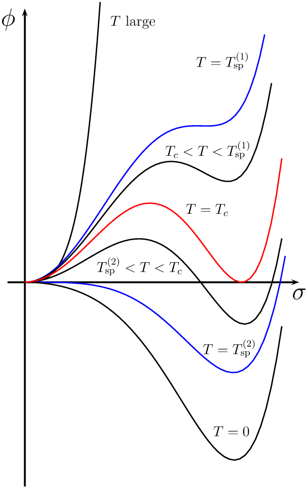


$\Rightarrow$ Large fluctuations in thermal systems!

First order phase transitions

- two degenerate minima separated by a barrier
- latent heat
- phase coexistence
- supercooling effects in nonequilibrium situations
- nucleation
- spinodal decomposition

(I.N.Mishustin, PRL **82** (1999); Ph.Chomaz, M.Colonna, J.Randrup, Physics Reports **389** (2004))



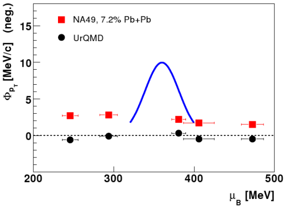
$\Rightarrow$ (Large) fluctuations in nonequilibrium situations!

Fluctuations at the critical point

non-monotonic fluctuations in pion and proton multiplicities

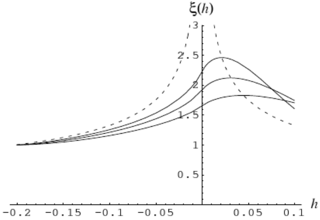
$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$

(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRD **60** (1999))



(NA49 collaboration J. Phys. G **35** (2008))

BUT: critical slowing down



(B. Berdnikov and K. Rajagopal, PRD **61** (2000))

Fluctuation measures based on the second moments are not conclusive about the critical behavior.

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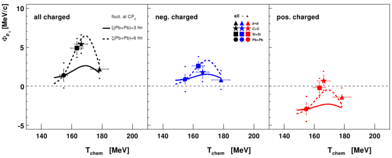

(NA49 collaboration, Acta Phys. Pol. **41** (2010))

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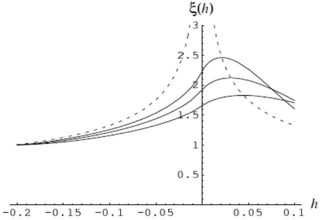

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Fluctuation measures based on the second moments are not conclusive about the critical behavior.

Definition of the kurtosis

susceptibilities of conserved charges (N: net-baryon, net-charge number) or the experimentally feasible net-proton number

$$\chi_n(T, \mu_N) = \frac{1}{VT^3} \frac{\partial^n \ln Z(V, T, \mu_N)}{\partial \mu_N^n}\bigg|_T$$

quadratic and quartic susceptibilities :

$$\chi_2 = \frac{1}{VT^3} \langle \delta N^2 \rangle$$
$$\chi_4 = \frac{1}{VT^3} (\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2)$$

effective kurtosis:

$$K^{\mathrm{eff}} = \frac{\chi_4}{\chi_2} = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle \equiv \kappa \sigma^2 \ .$$

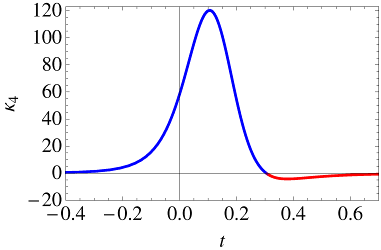
Higher moments at the critical point

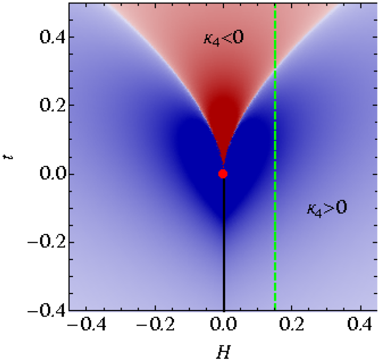
Higher moments of the distribution of conserved quantities are more sensitive to critical phenomena.

$$\kappa \propto \xi^7$$

(M. A. Stephanov, PRL **102**, 032301 (2009))

The kurtosis is negative at the critical point!

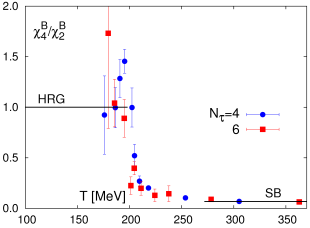


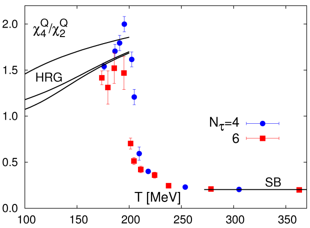


(M. A. Stephanov, PRL **107**, 052301, (2011))

Kurtosis on the lattice

Thermodynamic susceptibilities can be calculated on the lattice.





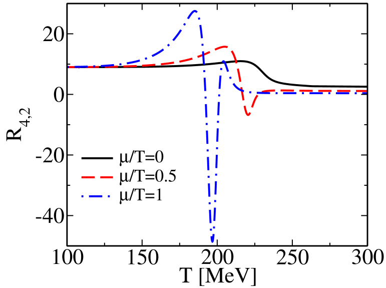
Reproduces the HRG below T_c and the Stefan-Boltzmann limit at high temperatures.
$\Rightarrow$ It has the potential to probe the confined and the deconfined phase of QCD.

M. Cheng *et al.*, Phys. Rev. D **79** (2009) 074505

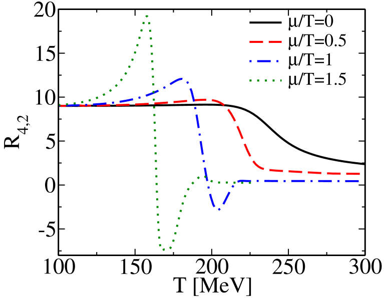
Kurtosis in effective models

The effective kurtosis can be calculated in effective models, e.g. in the Polyakov-loop extended quark-meson model:

in mean-field approximation



and in FRG

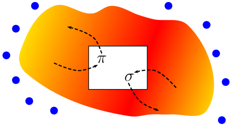


(V. Skokov, B. Stokic, B. Friman, K. Redlich, Phys. Rev. **C83** (2011) 054904.)

Motivation

- Fluctuations have so far been investigated in static systems.
- However, systems created in a heavy-ion collisions are finite in size and time and inhomogeneous.
- Necessary to propagate fluctuations explicitly!

- Nonequilibrium chiral fluid dynamics:
  - fluid dynamics +
  - phase transition model +
  - dissipation and noise



Nonequilibrium chiral fluid dynamics

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Fluid dynamic expansion of the quark fluid = heat bath

$$T_{\mathrm{q}}^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu}$$

- Energy and momentum exchange

$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu}$$

$\Longrightarrow$ Selfconsistent approach within the two-particle irreducible effective action!

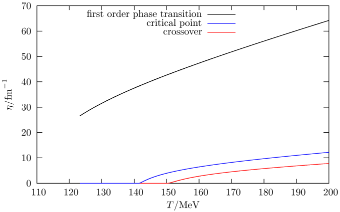
(MN, S. Leupold, C. Herold, M. Bleicher, PRC **84** (2011))

Semiclassical equation of motion for the sigma field

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

damping term η and noise ξ for $\mathbf{k} = 0$

$$\eta = g^2 \frac{d_q}{\pi} \left(1 - 2n_{\mathrm{F}}\left(\frac{m_\sigma}{2}\right)\right) \frac{\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{\frac{3}{2}}}{m_\sigma^2}$$

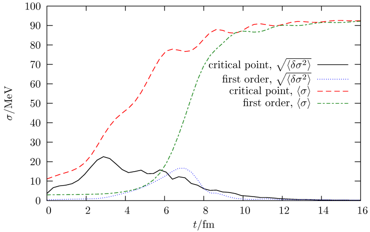


$$\langle \xi(t)\xi(t')\rangle = \frac{1}{V}\delta(t-t') m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)$$

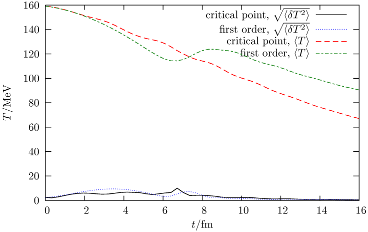
below T_c damping by the interaction with the hard pion modes, apply $\eta = 2.2/\mathrm{fm}$ from (T. S. Biro and C. Greiner, PRL **79** (1997))

Reheating and supercooling

relaxation of the σ field



temperature



- oscillations at the critical point
- supercooling of the system at the first order phase transition
- reheating effect visible at the first order phase transition

MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962

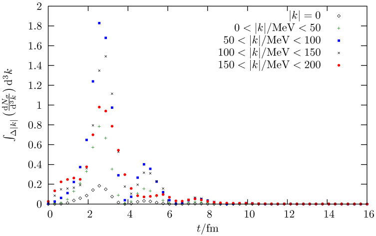
Intensity of sigma fluctuations

$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3k} = \frac{(\omega_k^2|\sigma_k|^2 + |\partial_t\sigma_k|^2)}{(2\pi)^3 2\omega_k}$$

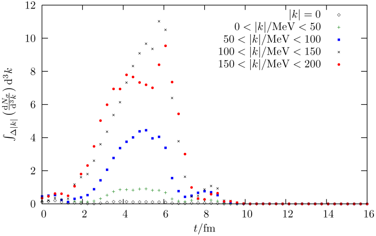
$$\omega_k = \sqrt{|k|^2 + m_\sigma^2}$$

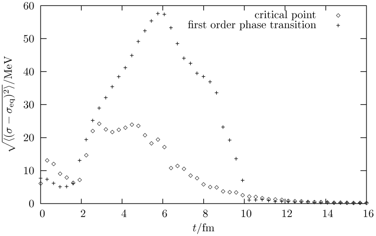
$$m_\sigma = \sqrt{\partial^2 V_{\mathrm{eff}}/\partial\sigma^2|_{\sigma=\sigma_{\mathrm{eq}}}}$$

critical point



first order phase transition





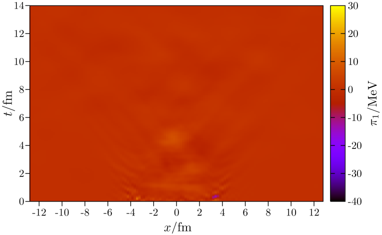
deviation from equilibrium

MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962

Pion fluctuations

So far: pion fluctuations were not considered and $\vec{\pi} = \langle \vec{\pi} \rangle = 0$.
Now: extend the model to explicitly propagate pion fluctuations, too.

critical point



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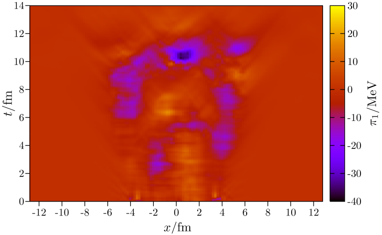
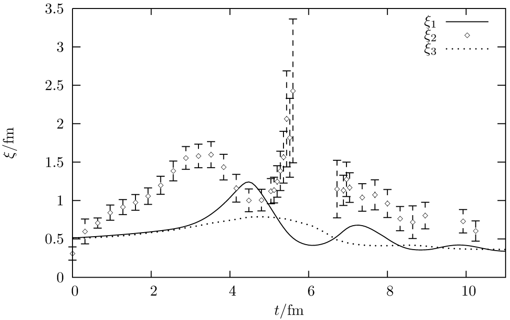


Larger isospin fluctuations in a scenario with a first order phase transition!

Correlation length



ξ_1: averaged correlation length from $\xi^{-1} = \sqrt{\frac{\partial^2 \Omega}{\partial \sigma^2}\Big|_{\sigma=\sigma(x)}}$

ξ_2: correlation length obtained from fits to $G(r) = \sigma_{\rm eq}^2 + \frac{1}{r}\exp(-\frac{r}{\xi})$

ξ_3: averaged correlation length from $\xi^{-1} = \sqrt{\frac{\partial^2 \Omega}{\partial \sigma^2}\Big|_{\sigma=\sigma_{\rm eq}}}$

Relativistic Transport Approach

cover more effects in realistic simulations of heavy-ion collisions, here: UrQMD (www.urqmd.org)

issues:

- eventwise baryon number and charge conservation instead of grandcanonical ensembles
- centrality selection and centrality bin width effects

Analytic toy model

Baryon number conservation limits fluctuations of net-baryon number.

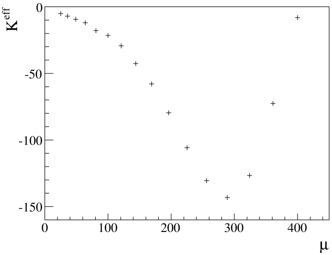
$$P_\mu(N, C) = \mathcal{N}(\mu, C) e^{-\mu} \frac{\mu^N}{N!} \quad \text{on} \quad [\mu - C, \mu + C]$$

μ: the expectation value of the original Poisson distribution, $\mathcal{N}(\mu, C)$: normalization factor, $C > 0$: cut parameter

$$C = \alpha \sqrt{\mu} \left(1 - \left(\frac{\mu}{N_{\mathrm{tot}}} \right)^2 \right).$$

$\alpha = 3$, $N_{\mathrm{tot}} = 416$.

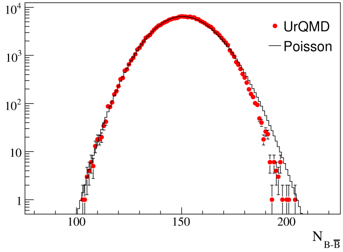
- An increase of the average net-baryon number does not lead to stronger fluctuations.
- At the upper limit of $N_{\mathrm{tot}} = 416$ the distribution changes to a δ-function ($K_\delta^{\mathrm{eff}} = 0$).

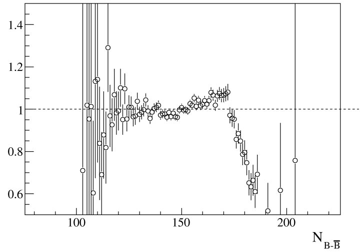


Net-baryon number distribution in UrQMD

- central Pb+Pb collisions at $E_{\mathrm{lab}} = 20A\mathrm{GeV}$
- fit to a Poisson distribution
- shoulders are enhanced
- tails are cut

$\Longrightarrow$ decrease from $K^{\mathrm{eff}}_{\mathrm{Poisson}} = 1$ to $K^{\mathrm{eff}}_{\mathrm{UrQMD}} = -22.2$



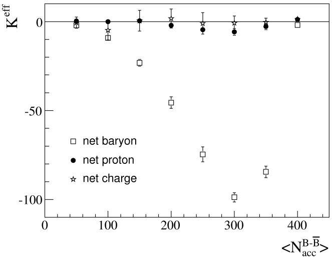


ratio of UrQMD to Poisson distribution

Rapidity window dependence of the effective kurtosis

- Same qualitative behavior of the net-baryon kurtosis as expected from the analytic toy model.

- $E_{\mathrm{lab}} = 158A\mathrm{GeV}$
- The net-proton kurtosis only slightly follows this trend.
- The net-charge kurtosis is not influenced, but error bars are larger.

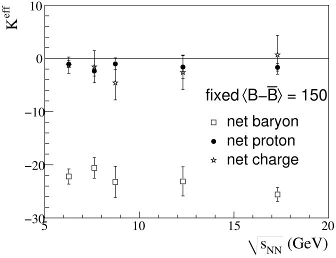


- For small net-baryon numbers in the acceptance, the values of net-baryon, net-proton and net-charge kurtosis are compatible with values of $0 - 1$.

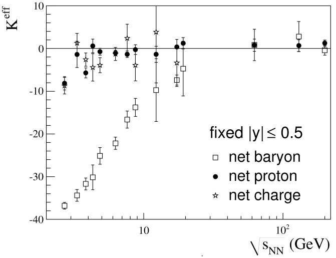
T. Schuster, MN, M. Mitrovski, R. Stock, M. Bleicher, [arXiv:0903.2911 [hep-ph]]

Energy dependence of the effective kurtosis

- adapting the rapidity window to fix the mean net-baryon number
- net-baryon effective kurtosis does not show an energy dependence



- fixed rapidity cut
- the net-baryon number varies with $\sqrt{s}$
- for lower $\sqrt{s}$ K^{eff} becomes increasingly negative
- at $E_{\mathrm{lab}} = 2A$GeV: $\langle N_{B-\bar{B}} \rangle \simeq 240$



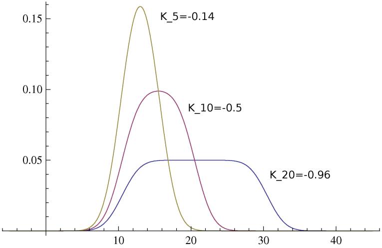
T. Schuster, MN, M. Mitrovski, R. Stock, M. Bleicher, [arXiv:0903.2911 [hep-ph]]

Centrality selection, e.g. by impact parameter

We investigate central collisions with $b \leq 2.75$ fm.
The superposition of two Gauss distributions (with mean $\mu_{1,2}$ and variance $\sigma_{1,2}$) has a negative kurtosis

$$K_2 = \frac{1/8\Delta\mu^4 + 3\Sigma^2\Delta\mu^2 + 6\Sigma^4}{1/8\Delta\mu^4 + \Sigma^2\Delta\mu^2 + 2\Sigma^4} - 3 < 0$$

with $\Delta\mu = |\mu_2 - \mu_1|$ and $\Sigma^2 = \sigma_1^2 + \sigma_2^2$.

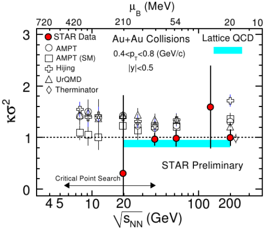


The distribution approaches a box-distribution with a $K_{\mathrm{box}} = -1.2$.

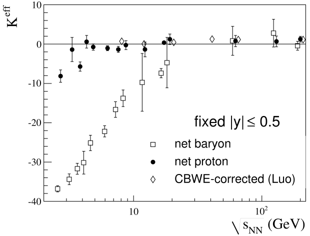
Effects of centrality selection

Suggestion by STAR to reduce centrality bin width effects:

- calculate moments for each fixed N_{charge} in one wider centrality bin
- take the weighted average



(STAR collaboration, Nucl.Phys. A862-863 (2011))



(MN et al., QM 2011 proceedings)

problems:

- (anti-) protons constitute a larger fraction of all charged particles with decreasing energy
- fixing N_{charge} puts a bias on the fluctuations

Summary

- nonequilibrium sigma and pion fluctuations, correlation length in chiral fluid dynamics
- effective kurtosis within a transport model of heavy-ion collisions
- negative values for net-baryon K^{eff} below $\sqrt{s} = 100$ GeV
- baryon number conservation qualitatively described by a cut Poisson distribution
- centrality selection remains a crucial issue!

Outlook:

- further study the acceptance effects in UrQMD
- extend nonequilibrium chiral fluid dynamics to study event-by-event fluctuations